

*Evidence-based Interventions for Students  
With Learning Disabilities:  
How Research Can Inform Practice*  
*Council for Learning Disabilities*

*October 3, 2009*

*David Chard*

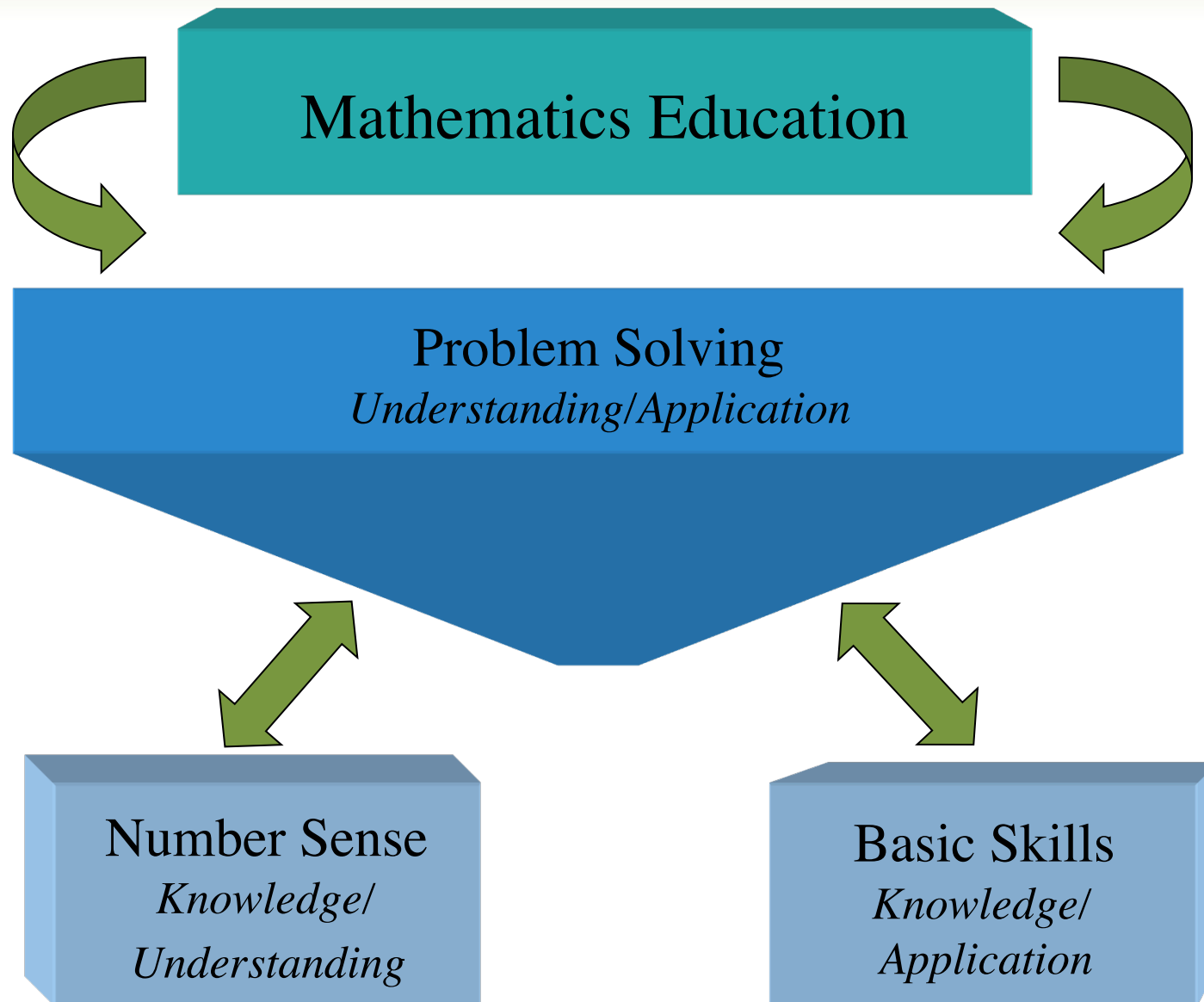
Dean of the Annette Caldwell Simmons School of Education &  
Human Development, Southern Methodist University

*Diane P. Bryant*

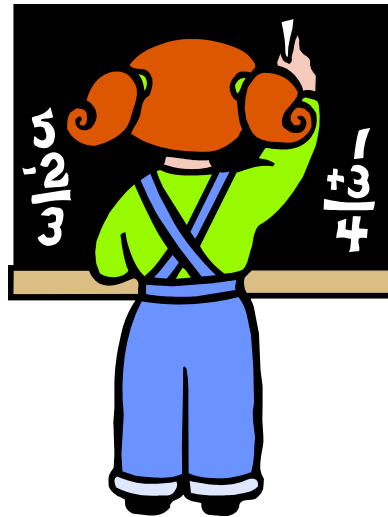
Project Director for the Meadows Center for Preventing Educational Risk:  
Mathematics Institute for Learning Disabilities & Difficulties

# Overview of the Session

- Discuss the context for evidence-based practices in mathematics education
- Describe a Response to Intervention (RtI) system for mathematics
- Describe findings of a meta-analysis for students with and at-risk for mathematics disabilities
- Describe an example of an evidence-based program designed to prevent mathematics disabilities



# Mathematics Difficulties



# What does it take . . . . .

- To use the count-on strategy to add  $9+3=?$   
(recognize +1, +2, +3 are count on strategies; min strategy: bigger # in head [9] count on 3 [keeping track of 3 while also counting consecutively - tap fingers, hold up fingers])
- To use the doubles +1 strategy?  
(Doubles, know doubles +1 = two numbers next to each other on the number line)
- To identify where to put the number 50 on a number line?  
(recognize start & end point; distance between; where 50 belongs)
- To use a hundreds chart to count by 10s beginning with 32?  
(start with 32, recognize 42 is 10 more)

# What does it take . . . . .

- To use the decomposition strategy to add  $9+4=?$   
( $9 + 1$  and  $4 = 3 + 1$ , use the 1 to make 10, now  $10 + 3 = 13$ ).
- To identify which number is greater: 49 or 62? 68 or 61?  
(start with 10s; go to ones)
- To tell which number comes before 21?  
(vocabulary: before, 20)
- To subtract two numbers that require regrouping?  
(understands place value, checks ones place, knows top number should be bigger, subtracts-knows facts)

# Common Difficulty Areas for Students with Mathematics Disabilities

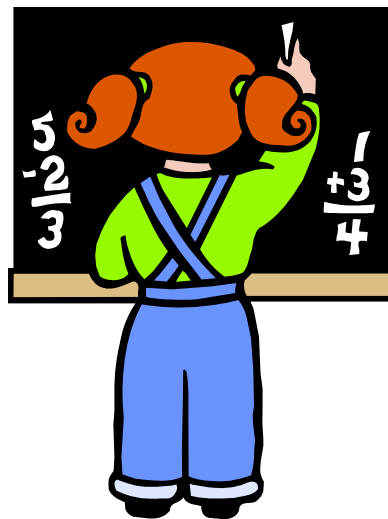
**Memory and  
Conceptual  
Difficulties**

**Background  
Knowledge  
Deficits**

**Linguistic and  
Vocabulary  
Difficulties**

**Difficulties with  
Strategy  
Knowledge  
and Use**

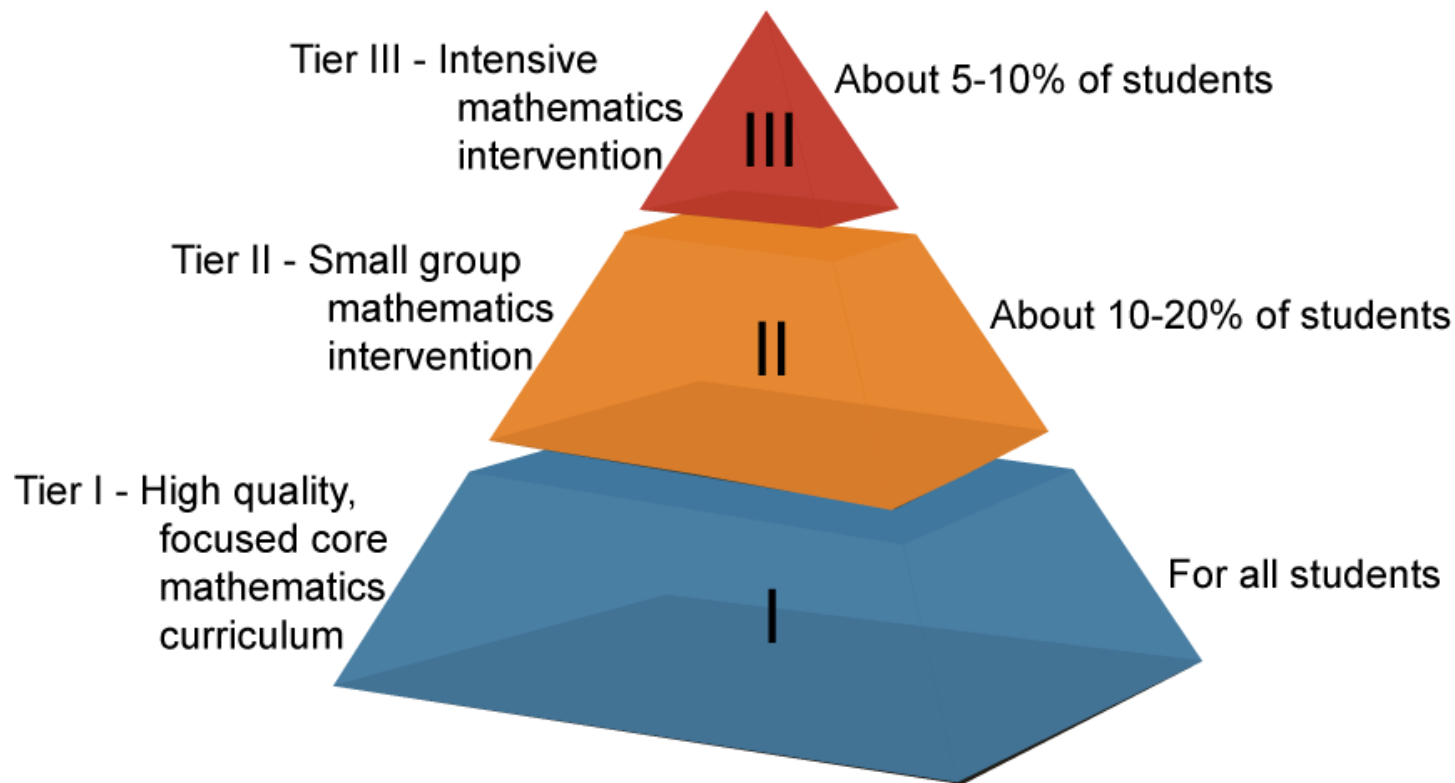
# Response to Intervention





# Example of a tiered model © Texas Education Agency

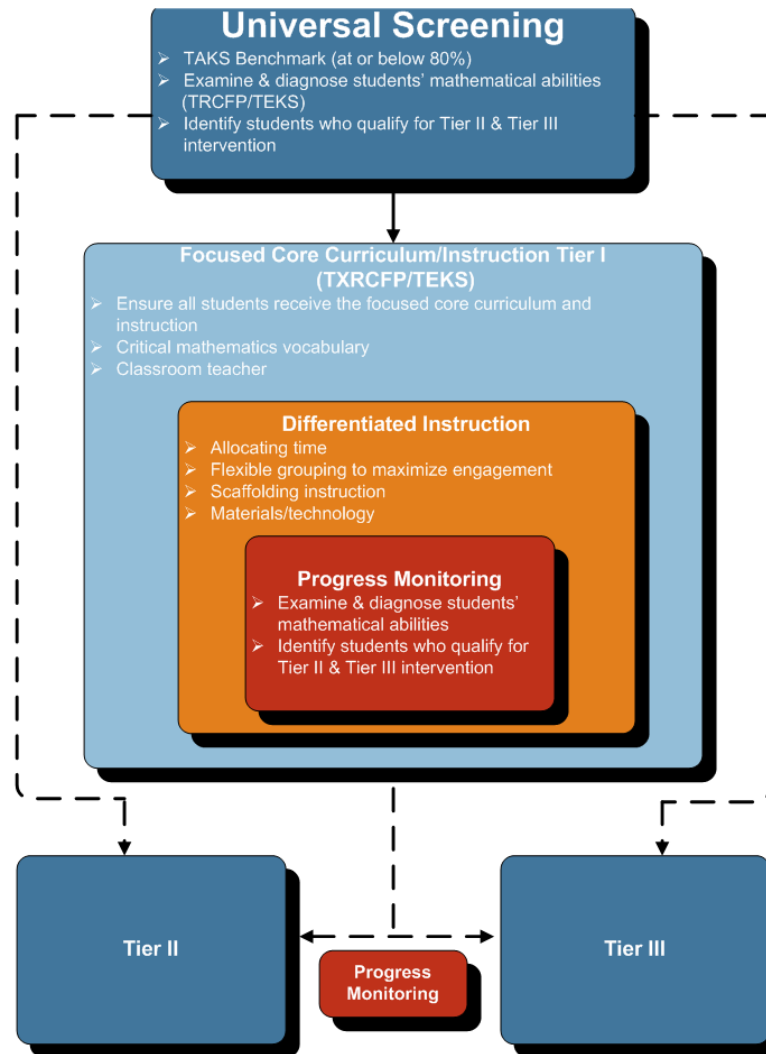
## 3 - Tier Intervention Model



# Example of Tier 1 or Core Instruction

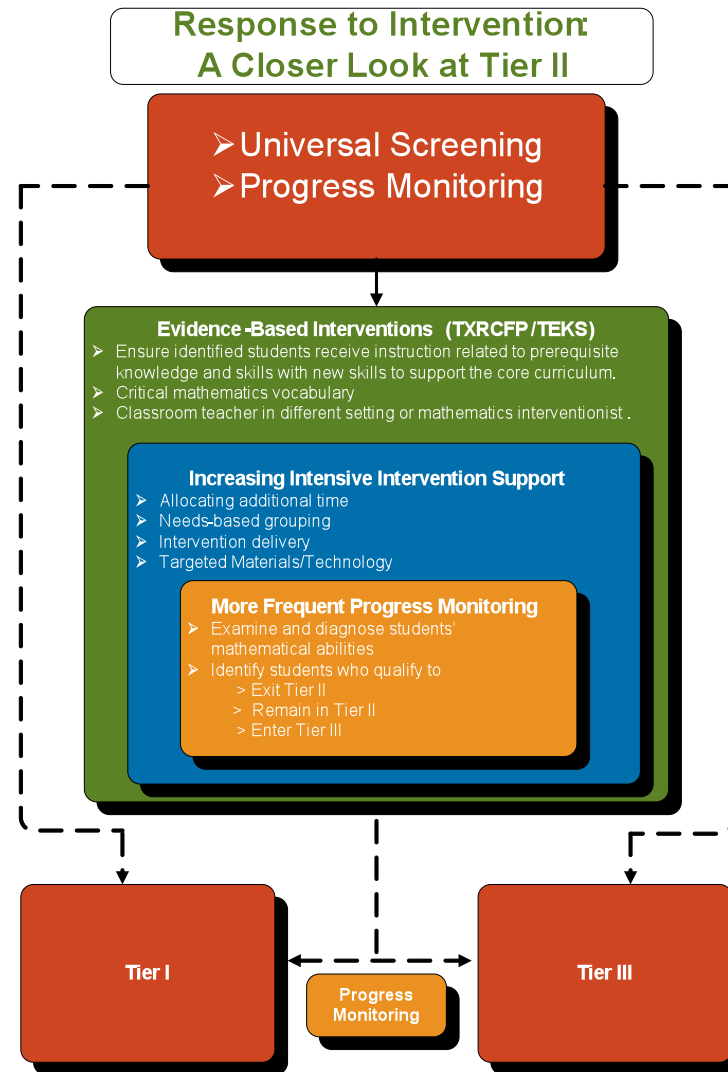
© Texas Education Agency

## Response to Intervention: Multi-tiered Model Possible Universal Screener: TAKS



# Example of Tier 2 Intervention

© Texas Education Agency





**MATHEMATICS INSTRUCTION FOR  
STUDENTS WITH LEARNING DISABILITIES  
OR DIFFICULTY LEARNING MATHEMATICS**

*A Guide for Teachers*



<http://centeroninstruction.org>

# Overview

- This booklet describes effective practices for students with mathematics difficulties (including learning disabilities).
- The meta-analysis including over 50 studies all of which employed randomized control trials or high quality quasi-experimental designs.

Gersten, R., Chard, D. J., Jayanthi, M., Baker, S. K., Morphy, P., & Flojo, J. (2009). Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components. *Review of Educational Research*, 79, 1202-1242.

# How were the effects of particular practices compared?

- The meta-analysis allows us to compare the relative effects of instructional practices using effect sizes.
- Effect sizes represent a proportion of a standard deviation.

Educationally  
Significant

$\geq .80$  large

$\geq .40$  moderate

$\geq .20$  small

$< .20$  extremely  
small/negligible

# Areas of Major Findings

- Use explicit instruction on a regular basis
- Employ multiple instructional examples
- Have students verbalize decisions and solutions to math problems
- Teach students to visually represent information in math problems
- Teach students to solve problems using multiple/heuristic strategies
- Provide ongoing formative assessment data and feedback to teachers
- Provide peer-assisted learning to students

## Effect Sizes for Instructional Variables

Instructional Recommendation	Number and Type of Studies Examined	Mean Effect Size (all Statistically Significant)
Explicit Instruction	10 Randomized Control Trials (RCT) 1 Quasi-experimental Designs (QED)	1.22 (Large)
Use Multiple Examples	9 RCTs	0.82 (Large)
Teach Students to Verbalize Decisions and Solutions	7 RCTs; 1 QED	1.04 (Large)
Teach Students to Visually Represent Information in Problems	11 RCTs; 1 QED	0.47 (Moderate)
Teach Students to Solve Problems using Multiple/Heuristic Strategies	3 RCTs; 1 QED	1.56 (Large)
Formative Assessment Data Provided to <i>Teachers</i>	10 RCTs	0.23 (Small)
Peer-assisted Learning	2 RCTs	1.02 (Large)



# Use explicit instruction on a regular basis

- Explicit instruction includes:
  - Clear modeling of the solution strategy to a problem
  - Thinking the specific steps aloud to a problem
  - Presenting multiple examples of a problem and their solutions
  - Providing immediate corrective feedback to students on their accuracy
- Explicit instruction should not be the whole of the teaching approaches used with any student, but must be used regularly with students who are experiencing mathematics difficulties.

# Teach students using multiple examples

- Spend time planning, focusing on the selection and sequence of examples
- Provide a wide range of examples of a problem type, highlighting problem variations but noting common and critical features
- Selection and sequencing of examples is particularly important during acquisition

# Have students verbalize decisions and solutions to math problems



- Encouraging students to think aloud the steps they use in solving a problem (specific and generic)
- Verbalizing steps in problem solving may address students' impulsivity and facilitate self-regulation in learning

# Teach students to visually represent the information in a math problem

- Graphic representations or drawings of problems and concepts are widely used
- Effects were enhanced when teachers taught students to select appropriate graphic representations and why a particular representation was most suitable
- This approach appears to be most beneficial when used by both teachers and students.

# Teach students to solve problems using multiple/heuristic strategies

- A heuristic strategy is a “generic” approach to solving a problem (e.g., read the problem, highlight relevant information, translate it into a math sentence, solve, check)
- Usually give students alternative approaches or options for solving the problem
- Typically involve teacher-led student discourse about the appropriateness of the solution chosen

# Provide formative assessment data to teachers

- Formative assessment is the process of collecting data on a randomly selected array of relevant topics at regular intervals (e.g. once per week or twice a month)
- Evidence has shown that this approach is superior to the typical weekly or biweekly unit tests that appear in many texts

# Provide formative assessment data to teachers

- Formative assessment use has consistently lead to low or moderate effects on mathematics achievement
- Feedback based on formative assessment coupled with specific suggestions for intervention strategies (e.g. problems for practice, alternate ways to explain a concept) improved effects
- This type of feedback was consistently effective for special education teachers.

# Engage students through peer-assisted learning

- Peer assisted learning provides extensive opportunities for students to practice solving math problems and to interact with peers about mathematics





# Engage students through peer assisted-learning

- Results have been consistently positive if:
  - Tutoring is provided by a proficient, trained peer
  - Student's work in pairs and the activities have a clear structure.
  - The pairs include students at differing ability levels.
  - Both students play the role of tutor for some of the time.
  - Students are trained in the procedures necessary to assume the role of tutor.

# Early Mathematics: RtI Prevention



# Early Predictors of Mathematics Achievement\*

Task	Areas Assessed
Counting skills	Set enumeration; rote counting; 1-1 correspondence; stable order; cardinality
Number knowledge	Relationships between numbers (e.g., magnitude comparisons)
Nonverbal calculation:	
Set transformations under a box	Adding or taking away objects hidden (“How many objects under the box?”).
Story problems	Single-digit addition and subtraction problems embedded in stories
Number combo	Single-digit addition and subtraction problems (“How much is $2 + 1$ ?”)

\*From: Jordan, N.C. (2007). Do words count? Compilation of results from several studies.

# Texas Early Mathematics Inventories-PM

**TEM I**

**Texas Early Mathematics Inventories**

Progress Monitoring  
FORM A

Student Name

Kindergarten TEM Progress Monitoring Form A  
© 2007 University of Texas System/Office of Education Research

**TEM I**

**Texas Early Mathematics Inventories**

Progress Monitoring  
FORM A

Student Name

First Grade TEM Progress Monitoring Form A  
© 2007 University of Texas System/Office of Education Research


**TEM 2**

**Texas Early Mathematics Inventories**

Progress Monitoring  
FORM A

Student Name

Second Grade TEM Progress Monitoring Form A  
© 2007 University of Texas System/Office of Education Research



**Number Sequences**


Demonstrations

2	3	__
1	3	
5	4	

19	__	21
20	19	
17	11	

__	54	55
53	52	
55	63	

1



**Practice**

1	2	__
1	3	
5	13	

16	__	18
5	19	
17	21	

__	81	82
19	79	
83	80	

6	__	8
7	5	
3	10	

__	1	2
3	4	
0	11	

3	4	__
11	2	
5	6	


  

__	8	9
1	4	
7	0	

1	2	__
5	0	
3	13	

13	14	__
14	15	
2	1	

2



5	__	7
6	8	
9	18	

__	4	5
2	6	
44	3	

5	6	__
9	4	
7	57	

0	__	2
3	20	
5	1	

__	14	15
13	16	
12	23	

12	13	__
11	25	
35	14	

18	19	__
17	20	
30	41	

17	18	__
21	25	
19	30	

19	20	__
21	27	
16	30	

3

- Subtests: MC, NS, ASC, PV or Rel of 10; NI, QR
- All TEMI-PM tests have three sections.
1. Demonstrations
  2. Practice
  3. Test items

# Texas Early Mathematics Inventories-O

**TEMI K** **Texas Early Mathematics Inventories**

Outcome  
FORM A

Student Name

© 2009 Meadows Center for Preventing Educational Risk

**TEMI I** **Texas Early Mathematics Inventories**

Outcome  
FORM A

Student Name

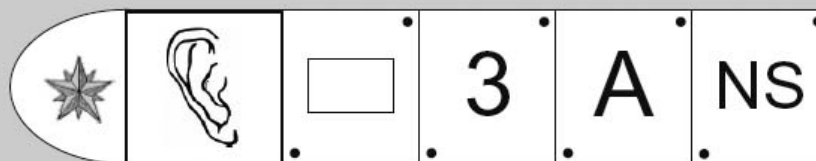
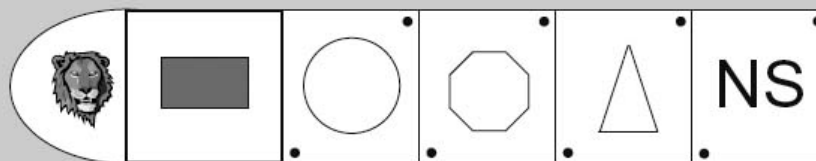
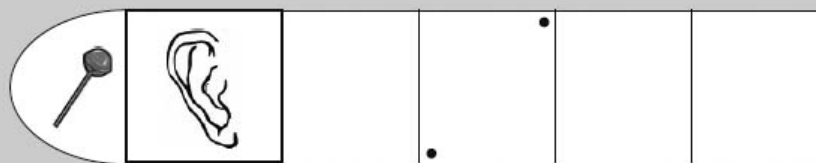
© 2009 Meadows Center for Preventing Educational Risk

**TEMI 2** **Texas Early Mathematics Inventories**

Outcome  
FORM A

Student Name

© 2009 Meadows Center for Preventing Educational Risk





$4 + 0 =$	$5 + 1 =$	$\begin{array}{r} 5 \\ + 2 \\ \hline \end{array}$																		
<table border="1"><tr><td>0</td><td>40</td></tr><tr><td>3</td><td>4</td></tr><tr><td colspan="2">NS</td></tr></table>	0	40	3	4	NS		<table border="1"><tr><td>6</td><td>4</td></tr><tr><td>7</td><td>5</td></tr><tr><td colspan="2">NS</td></tr></table>	6	4	7	5	NS		<table border="1"><tr><td>7</td><td>3</td></tr><tr><td>8</td><td>10</td></tr><tr><td colspan="2">NS</td></tr></table>	7	3	8	10	NS	
0	40																			
3	4																			
NS																				
6	4																			
7	5																			
NS																				
7	3																			
8	10																			
NS																				
$\begin{array}{r} 6 \\ + 3 \\ \hline \end{array}$	$6 - 0 =$	$\begin{array}{r} 60 \\ + 20 \\ \hline \end{array}$																		
<table border="1"><tr><td>6</td><td>9</td></tr><tr><td>10</td><td>3</td></tr><tr><td colspan="2">NS</td></tr></table>	6	9	10	3	NS		<table border="1"><tr><td>0</td><td>6</td></tr><tr><td>5</td><td>60</td></tr><tr><td colspan="2">NS</td></tr></table>	0	6	5	60	NS		<table border="1"><tr><td>8</td><td>90</td></tr><tr><td>80</td><td>40</td></tr><tr><td colspan="2">NS</td></tr></table>	8	90	80	40	NS	
6	9																			
10	3																			
NS																				
0	6																			
5	60																			
NS																				
8	90																			
80	40																			
NS																				
$\begin{array}{r} 52 \\ + 35 \\ \hline \end{array}$	$18 - 0 =$	$\begin{array}{r} 4 \\ + 6 \\ \hline \end{array}$																		
<table border="1"><tr><td>15</td><td>87</td></tr><tr><td>137</td><td>86</td></tr><tr><td colspan="2">NS</td></tr></table>	15	87	137	86	NS		<table border="1"><tr><td>0</td><td>18</td></tr><tr><td>10</td><td>17</td></tr><tr><td colspan="2">NS</td></tr></table>	0	18	10	17	NS		<table border="1"><tr><td>12</td><td>10</td></tr><tr><td>2</td><td>9</td></tr><tr><td colspan="2">NS</td></tr></table>	12	10	2	9	NS	
15	87																			
137	86																			
NS																				
0	18																			
10	17																			
NS																				
12	10																			
2	9																			
NS																				



$\begin{array}{r} 5 \\ 3 \\ + 1 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ - 2 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ - 3 \\ \hline \end{array}$																		
<table border="1"><tr><td>9</td><td>5</td></tr><tr><td>8</td><td>45</td></tr><tr><td colspan="2">NS</td></tr></table>	9	5	8	45	NS		<table border="1"><tr><td>3</td><td>9</td></tr><tr><td>10</td><td>2</td></tr><tr><td colspan="2">NS</td></tr></table>	3	9	10	2	NS		<table border="1"><tr><td>9</td><td>0</td></tr><tr><td>6</td><td>4</td></tr><tr><td colspan="2">NS</td></tr></table>	9	0	6	4	NS	
9	5																			
8	45																			
NS																				
3	9																			
10	2																			
NS																				
9	0																			
6	4																			
NS																				
$\begin{array}{r} 6 \\ + 6 \\ \hline \end{array}$	$\begin{array}{r} 10 \\ - 4 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ + 8 \\ \hline \end{array}$																		
<table border="1"><tr><td>0</td><td>8</td></tr><tr><td>4</td><td>10</td></tr><tr><td colspan="2">NS</td></tr></table>	0	8	4	10	NS		<table border="1"><tr><td>5</td><td>6</td></tr><tr><td>14</td><td>3</td></tr><tr><td colspan="2">NS</td></tr></table>	5	6	14	3	NS		<table border="1"><tr><td>16</td><td>15</td></tr><tr><td>12</td><td>1</td></tr><tr><td colspan="2">NS</td></tr></table>	16	15	12	1	NS	
0	8																			
4	10																			
NS																				
5	6																			
14	3																			
NS																				
16	15																			
12	1																			
NS																				
$\begin{array}{r} 8 \\ + 9 \\ \hline \end{array}$	$\begin{array}{r} 720 \\ + 115 \\ \hline \end{array}$	$\begin{array}{r} 800 \\ - 400 \\ \hline \end{array}$																		
<table border="1"><tr><td>4</td><td>9</td></tr><tr><td>0</td><td>7</td></tr><tr><td colspan="2">NS</td></tr></table>	4	9	0	7	NS		<table border="1"><tr><td>835</td><td>610</td></tr><tr><td>830</td><td>805</td></tr><tr><td colspan="2">NS</td></tr></table>	835	610	830	805	NS		<table border="1"><tr><td>400</td><td>900</td></tr><tr><td>1200</td><td>40</td></tr><tr><td colspan="2">NS</td></tr></table>	400	900	1200	40	NS	
4	9																			
0	7																			
NS																				
835	610																			
830	805																			
NS																				
400	900																			
1200	40																			
NS																				





# Identification/Diagnosis

**Fall and Winter: Identify  
Struggling Students**

<b>&lt;25<sup>th</sup> %ile</b>	<b>25<sup>th</sup> through 35<sup>th</sup> %ile</b>	<b>&gt;35<sup>th</sup> %ile</b>
---------------------------------	---	---------------------------------

**Total Score = Sum of TEMI-PM raw scores; compare  
to Benchmark**

**A B C**

**A = All ahead full (>35<sup>th</sup> %ile)**

**B = Be alert (25<sup>th</sup> through 35<sup>th</sup> %ile)**

**C = Change! (<25<sup>th</sup> %ile)**

**Also able to  
establish EXIT  
criteria!**

# Progress Monitoring


## Step 1. Insert student TEMI-PM Total Scores.

	MC			NS			PV			ASC			Total Score		
	Autumn	Winter	Spring	Autumn	Winter	Spring	Autumn	Winter	Spring	Autumn	Winter	Spring	Autumn	Winter	Spring
Student A	1	32	47	11	18	20	3	14	18	1	14	19	16	78	104
Student B	19	25	18	9	11	11	6	14	10	4	4	14	38	54	53
Student C	31	37	39	10	20	22	8	15	18	8	11	18	57	83	97
Student D	35	32	41	13	22	24	6	13	22	4	11	13	58	78	100
Student E	4	17	42	2	5	14	3	6	12	2	4	12	11	32	80
Student F	10	23	25	2	7	16	1	6	15	2	4	10	15	40	66
Student G	1	28	36	1	16	17	3	8	17	0	6	11	5	58	81
Student H	32	44	46	18	27	27	8	23	24	10	15	29	68	109	126
Student I	40	40	49	13	20	25	7	11	16	11	13	20	71	84	110
Student J	53	53	51	21	28	25	3	17	23	3	19	19	80	117	118
Student K	44	36	42	8	22	25	10	15	21	10	14	18	72	87	106
Student L	42	37	39	7	21	12	7	14	14	5	11	14	61	83	79
Student M	36	40	32	6	20	20	7	12	20	9	5	18	58	77	90
Student N	29	31	28	14	19	19	7	18	19	4	9	15	54	77	81
Student O	20	37	38	11	17	13	4	15	18	3	8	17	38	77	86
Student P	61	54	57	40	30	27	25	21	24	16	24	24	142	129	132
Student Q	1	32	47	11	18	20	3	14	18	1	14	19	16	78	104
Student S	19	25	18	9	11	11	6	14	10	4	4	14	38	54	53
Student T	31	37	39	10	20	22	8	15	18	8	11	18	57	83	97
Student U													0	0	0
Student V													0	0	0
Class Averages	27	35	39	11	19	19	7	14	18	6	11	17	45	70	84
Total Students Tested	21	21	21	21	21	21	21	21	21	21	21	21	23	23	23
Number of Students Below 25th Percentile	5	4	4	4	4	5	7	2	3	3	6	3	7	7	5
% of Students Below 25th Percentile	24%	19%	19%	19%	19%	24%	33%	10%	14%	14%	29%	14%	30%	30%	22%
Spring 25th Percentile Line	27		32	11		16	7		14	6		13	45		77

# Texas Early Mathematics Inventories

## Classroom Report - Winter

TEMI Progress Monitoring, 1st Grade

<b>School:</b> Sample <b>Teacher:</b> Example			<b>Fall 2008</b> <i>Benchmark = 49</i>		<b>Winter 2009</b> <i>Benchmark = 60</i>		<b>Spring 2009</b> <i>Benchmark = 96</i>	
	Score	Classification	Score	Classification	Score	Classification	Score	Classification
Student A	87	A	89	A				
Student B	33	C	61	B				
Student C	49	B	Absent	---				
Student D	89	A	98	A				
Student E	33	C	83	A				
Student F	19	C	90	A				
Student G	Absent	---	123	A				
Student H	46	C	90	A				
Student I	98	A	50	C				
Student J	23	C	20	C				
Student K	Absent	---	63	B				
Student L	22	C	100	A				
Student M	11	C	23	C				
Student N	28	C	150	A				
Student O	No data	---	98	A				

Classification key: A = All Ahead Full (above 35th percentile on the TEMI-PM Total Score)

B = Be Aware (from the 25th through the 35th percentile on the TEMI-PM Total Score)

C = Change Needed (below 25th percentile on the TEMI-PM Total Score)

# Intervention

## The conceptual development routine

- Review prerequisite skills/background knowledge (e.g., warm up).
- Modeled practice, paired with teacher-guided practice to engage students in solving problems.
- Provide multiple examples based on student needs.
- Provide distributive practice.
- Scaffold instruction as needed (e.g., using think alouds, breaking down difficult tasks into additional instructional steps, providing more explanations).
- Maintain an appropriate pace that reflects the instructional needs of students (e.g., slowing if material is difficult).
- Engage students throughout the lesson with multiple opportunities to respond (verbal, written, hands-on).
-

# Intervention

## The conceptual development routine

- Ask students to explain their answers.
- Use manipulatives and/or representations (pictorial, abstract, symbolic) to model concepts and skills.
- Provide error correction procedures to correct mistakes and to provide feedback.
- Teach efficient strategies (e.g., doubles + 1, derived strategies) that facilitate student understanding and learning.
- Emphasize the mathematics vocabulary of the lesson.
- Check student understanding throughout the lesson.
- Monitor student progress to make data-based instructional decisions about student performance with the intervention.

# Intervention Components

## Units/Lessons

- 10 units, 8 lessons per unit
- Daily components: warm-up (review-facts, writing numbers), 2 lessons, cool down

## Grouping

- Homogeneous grouping with 4 - 5 students per group

## Duration/Length

- 21 weeks; 4 days per week; 30 min.

## Representations

- Physical (concrete), visual (pictorial), abstract (numbers and symbols)
- 100s chart, 5- and 10-frames, counters, number lines, base-ten materials, fact cards

## Progress Monitoring

- Daily checks (lessons for the day)
- Unit checks (multi skills from the unit)
- Aim Checks (fluency)

# Intervention

## Instructional Content (Examples)

### **Number Knowledge and Relationships**

- Count: Rote, Counting Up/Back, Skip (2, 5, 10)
- Read & write numbers: 0 – 99
- Compare & order numbers and magnitude of numbers

### **Relationships of 10**

- Use models to represent numbers: groups of tens and ones
- Create equivalent representations of numbers
- Compose and decompose numbers - multi-digit numbers

### **Addition & Subtraction Combinations**

- Identify and apply properties
- Develop and apply strategies to solve facts (e.g., count on/back doubles, doubles +1, make 10 + more)
- Solve addition & related subtraction problems



## Warm-Up: Number Recognition

**Directions:** Hold up number cards and have students say each number with a quick oral response (within 3–4 seconds). If students say an incorrect number for a card, put it in a pile for extra practice. After students go through all the number cards, review the cards in the extra-practice pile and tell students to repeat the correct answers.



### Time:

Set a timer for 2 minutes.  
Allow enough time to go  
over incorrect answers.

### Materials:

Number Cards (0-20)





# Warm-Up Example

## Units 1 & 2 Warm-Up

1	1	1	0	1	0	4	0	1	5	8	
	<u>+0</u>	<u>+1</u>	<u>+0</u>	<u>+9</u>	<u>+3</u>	<u>+1</u>	<u>+6</u>	<u>+7</u>	<u>+0</u>	<u>+1</u>	<u>10</u>

2	5	0	3	0	1	7	0	6	4	0	
	<u>+1</u>	<u>+8</u>	<u>+1</u>	<u>+1</u>	<u>+2</u>	<u>+0</u>	<u>+9</u>	<u>+1</u>	<u>+0</u>	<u>+2</u>	<u>10</u>

3	0	2	5	1	2	0	6	8	1	2	
	<u>+0</u>	<u>+9</u>	<u>+0</u>	<u>+2</u>	<u>+2</u>	<u>+3</u>	<u>+1</u>	<u>+2</u>	<u>+7</u>	<u>+3</u>	<u>10</u>

4	0	3	2	1	5	0	2	4	6	9	
	<u>+2</u>	<u>+1</u>	<u>+4</u>	<u>+0</u>	<u>+2</u>	<u>+8</u>	<u>+7</u>	<u>+1</u>	<u>+2</u>	<u>+0</u>	<u>10</u>

## My Progress

10				
9				
8				
7				
6				
5				
4				
3				
2				
1				
	1	2	3	4

# Addition and Subtraction Facts

- Solve basic addition and subtraction facts
- Solve facts fluently
- Use strategies (Count on, Count down, Doubles, Doubles +1, Make 10 plus more)

# Addition Subtraction Combinations

**Objective:** The student will be able to use concrete objects, pictures, and the Make 10 and 10 + More strategies to solve addition facts.

**Instructional Content:**

Addition facts to 17 (Make 10 + More)

**Vocabulary**

Add, equals, plus, strategy, turnaround fact

**Materials:**

Addition facts to 17  
(Make 10 + More)

15 Counters (cubes, anything to use as markers) (T)

## Modeled Practice

Unit 1  
Booster Lesson 1  
K3C Op 1  
Modeled Practice  
Setup Time 1

A ten-frame with 9 dots in the top row and 1 dot in the bottom row. To the left of the ten-frame is a plus sign and a box for the answer.

## Guided Practice

Unit 1  
Booster Lesson 1  
K3C Op 1  
Guided Practice  
Setup Time 1

A grid of ten-frames with dots and plus signs, illustrating various addition facts. The facts shown are: 9 + 1 = 10, 8 + 2 = 10, 7 + 3 = 10, 6 + 4 = 10, 5 + 5 = 10, 4 + 6 = 10, 3 + 7 = 10, 2 + 8 = 10, 1 + 9 = 10, and 10 + 0 = 10.

## Independent Practice

Unit 1  
Booster Lesson 1  
K3C Op 1  
Independent Practice  
Setup Time 1

A grid of ten-frames with dots and plus signs, illustrating various addition facts. The facts shown are: 9 + 1 = 10, 8 + 2 = 10, 7 + 3 = 10, 6 + 4 = 10, 5 + 5 = 10, 4 + 6 = 10, 3 + 7 = 10, 2 + 8 = 10, 1 + 9 = 10, and 10 + 0 = 10.

*Today we will learn a strategy to add numbers up to 17.*

*We will use Make 10.* (Review:  $7 + \underline{\quad} = 10$ ,  $9 + \underline{\quad} = 10$ )

*We will use 10 + More.* (Review:  $10 + 1 \text{ more} = \underline{\quad}$ ,  $10 + 3 \text{ more} = \underline{\quad}$ , etc.)

*Together, these strategies make the Make 10 + More strategy.*

## Modeled Practice

1. Place the Modeled Practice Sheet on the table. Have students look at their Modeled Practice Sheets. Introduce the Make 10 + More strategy using the fact:  $9 + 4$ .  $9 + 4 = ?$

**My Turn:** *I put a group of 9 counters in the top frame to show 9, Ready count: 1, 2, 3 . . . 9 and a group of 4 counters in the bottom frame to show + 4. Ready count: 1, 2, 3, 4.. (Cover each circle with a counter).*

**Your Turn:** *Try it.* (Repeat above steps with students)

**My Turn:** *When I see a fact with 7, 8, or 9, I use both strategies we just learned: Make 10 and 10 + More.*

*We call the strategy "Make 10 + More."*

*There are 3 steps.*

*Step 1: Check the fact; is there a 7, 8, or 9 in it? (yes)*

*There is a 9 in this fact.*

*Step 2: Make 10.*

*9 plus what equals 10? (1)*

*I take one counter from the group of four to put with the group of nine. (Move the counter over the dotted arrow to the dotted*

**Your Turn:** *Try it. Make 10.* (Repeat above steps with students.)

*Step 3: Add 10 + More.*

*We have 10 in one frame, plus 3 remaining counters.*

*Start with 10 and count on. Ready count: 10, 11, 12, 13.*

*What is  $10 + 3$ ? (13) So,  $9 + 4 = 13$ . Write it.*

*What is the turnaround fact? ( $4 + 9 = 13$ )*

*Why does  $10 + 3$  equal  $9 + 4$ ? (Encourage students to think about how the Make 10 + More strategy helps solve hard facts)*

2. Clean the counters off the sheet. Complete the same process with students, drawing circles instead of using counters. Show and say the fact card  $9 + 4$ .

**Your Turn:** *Now use pictures to solve the fact.*

*There are 9 circles in the top frame and 4 circles in the bottom frame to show  $9 + 4$ .*

*One of the bottom circles is dotted to remind me to Make 10.*

**My Turn:** *Step 1: Check the fact; is there a 7, 8, or 9 in it? (yes)*

*There is a 9 in this fact.*

*Step 2: Make 10.*

*9 plus what number equals 10? (1)*

*I cross out the dotted circle from the group of 4 and add that circle to the group of 9. It's like I moved it!*

*I know that  $9 + 1 = 10$ . I made 10!*

**Your Turn:** *Try it. Make 10.* (Repeat above steps with students.)



### Diagnosis and Correction

The student has difficulty drawing circles in a double ten-frame format.

- » 1 phrase
- » 1 phrase
- » 1 phrase

3. Give students the Guided Practice sheets. Use the following language in each problem and go through the sheet with students. Note: It is very important to go through each step listed below with students so they can learn the steps through repetition.

**My Turn:**

*Step 1: Check the fact; is there a 7, 8, or 9?*  
(yes)

*How many circles are in the top ten frame?* (9)

*How many circles are in the bottom ten frame?* (5)

*Step 2: Make 10.*

*9 plus what equals 10?* (1; Cross out the dotted circle and move circle to top ten frame.)

*Step 3: Add 10 + more.*

*How many remaining circles?* (4)

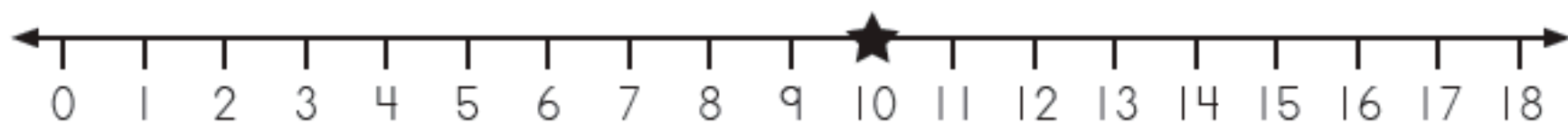
*What is the answer to  $10 + 4$ ?* ( $10 + 4 = 14$ )

*What is the answer to the fact  $9 + 5$  then?* ( $9 + 5 = 14$ )

*Write it.*

*Why does  $10 + 4 = 9 + 5$ ?* (Have students explain why  $10 + 4 = 9 + 5$ .)





<p>1</p> $\begin{array}{r} 4 \\ + 4 \\ \hline \end{array}$	<p>2</p> $4 + 5 =$	<p>3</p> $\begin{array}{r} 5 \\ + 5 \\ \hline \end{array}$	<p>4</p> $5 + 6 =$
<p>5</p> $\begin{array}{r} 8 \\ + 8 \\ \hline \end{array}$	<p>6</p> $\begin{array}{r} 8 \\ + 9 \\ \hline \end{array}$	<p>7</p> $7 + 7 =$	<p>8</p> $\begin{array}{r} 7 \\ + 8 \\ \hline \end{array}$
<p>9</p> $5 + 4 =$	<p>10</p> $\begin{array}{r} 6 \\ + 7 \\ \hline \end{array}$	<p>11</p> $9 + 8 =$	<p>12</p> $8 + 7 =$

# Addition and Subtraction Facts

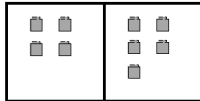


Doubles +1

Unit 8 • Lesson 5  
ASF • GP

1

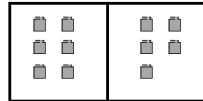
$$4 + 4 = \underline{\quad}$$



$$4 + 5 = \underline{\quad}$$

2

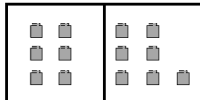
$$5 + 5 = \underline{\quad}$$



$$6 + 5 = \underline{\quad}$$

3

$$6 + 6 = \underline{\quad}$$



$$6 + 7 = \underline{\quad}$$

4

$$7 + 7 = \underline{\quad}$$



$$7 + 8 = \underline{\quad}$$

5

$$8 + 8 = \underline{\quad}$$



$$9 + 8 = \underline{\quad}$$

6

$$9 + 9 = \underline{\quad}$$



$$10 + 9 = \underline{\quad}$$



Count-Back

Unit 5 • Lesson 23  
ASF • IP



1

$$\begin{array}{r} 9 \\ - 3 \\ \hline \end{array}$$

2

$$1 - 1 = \underline{\quad}$$

3

$$6 - 3 = \underline{\quad}$$

4

$$\begin{array}{r} 4 \\ - 3 \\ \hline \end{array}$$

5

$$\begin{array}{r} 7 \\ - 7 \\ \hline \end{array}$$

6

$$5 - 3 = \underline{\quad}$$

7

$$\begin{array}{r} 10 \\ - 3 \\ \hline \end{array}$$

8

$$\begin{array}{r} 7 \\ - 3 \\ \hline \end{array}$$

9

$$4 - 4 = \underline{\quad}$$

10

$$8 - 3 = \underline{\quad}$$



# Ordering and Comparing Numbers

- Order numbers from least to greatest
- Identify missing numbers in a number sequence using strategies
- Use patterns to count (skip counting)

# Ordering & Comparing Numbers: Student Work Materials



Neighbor, Neighbor

Unit 2 • Lesson 3  
OCN • GP

101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

- 1 Before: \_\_\_\_ 184
- 2 After: 108 \_\_\_\_
- 3 Between: 197 \_\_\_\_ 199
- 4 Before and After: \_\_\_\_ 115 \_\_\_\_
- 5 Between: 136 \_\_\_\_ 138
- 6 After: 159 \_\_\_\_
- 7 Before: \_\_\_\_ 114
- 8 Between: 144 \_\_\_\_ 146
- 9 After: 175 \_\_\_\_
- 10 Before and After: \_\_\_\_ 179 \_\_\_\_



Neighbor, Neighbor

Unit 2 • Lesson 3  
OCN • 1P

101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

- 1 Between: 121 \_\_\_\_ 123
- 2 After: 187 \_\_\_\_
- 3 Before: \_\_\_\_ 145
- 4 Before and After: \_\_\_\_ 120 \_\_\_\_
- 5 Between: 134 \_\_\_\_ 136
- 6 After: 129 \_\_\_\_
- 7 After: 169 \_\_\_\_
- 8 Before: \_\_\_\_ 130
- 9 Between: 185 \_\_\_\_ 187
- 10 Before and After: \_\_\_\_ 156 \_\_\_\_

# Relationships of Ten

- Identify a number represented by picture or objects
- Identify the “ones place,” “tens place,” and “hundreds place”
- Identify greater than and less than using pictures or objects and place value

# Relationships of Ten



Add 'Em

Unit 8 • Lesson 12  
ROT • GP

				Add Together + _____
23	7	8	26	
43	35	18	62	
				+ _____
37	13	31	4	
56	67	20	25	
				+ _____
80	8	15	4	
40	50	25	5	



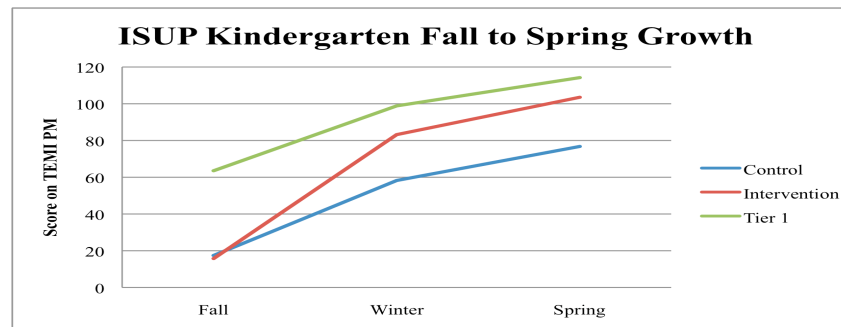
Adding BIG Numbers!

Unit 8 • Lesson 12  
ROT • IP

<p>1</p> <table border="1"> <tr><th>Tens</th><th>Ones</th></tr> <tr><td></td><td></td></tr> <tr><td>+</td><td></td></tr> <tr><td>_____</td><td>_____</td></tr> </table>	Tens	Ones			+		_____	_____	<p>2</p> <table border="1"> <tr><th>Tens</th><th>Ones</th></tr> <tr><td></td><td></td></tr> <tr><td>+</td><td></td></tr> <tr><td>_____</td><td>_____</td></tr> </table>	Tens	Ones			+		_____	_____
Tens	Ones																
+																	
_____	_____																
Tens	Ones																
+																	
_____	_____																
<p>3</p> <table border="1"> <tr><th>Tens</th><th>Ones</th></tr> <tr><td></td><td></td></tr> <tr><td>+</td><td></td></tr> <tr><td>_____</td><td>_____</td></tr> </table>	Tens	Ones			+		_____	_____	<p>4</p> <table border="1"> <tr><th>Tens</th><th>Ones</th></tr> <tr><td></td><td></td></tr> <tr><td>+</td><td></td></tr> <tr><td>_____</td><td>_____</td></tr> </table>	Tens	Ones			+		_____	_____
Tens	Ones																
+																	
_____	_____																
Tens	Ones																
+																	
_____	_____																
<p>5</p> <table border="1"> <tr><th>Tens</th><th>Ones</th></tr> <tr><td></td><td></td></tr> <tr><td>+</td><td></td></tr> <tr><td>_____</td><td>_____</td></tr> </table>	Tens	Ones			+		_____	_____	<p>6</p> <table border="1"> <tr><th>Tens</th><th>Ones</th></tr> <tr><td></td><td></td></tr> <tr><td>+</td><td></td></tr> <tr><td>_____</td><td>_____</td></tr> </table>	Tens	Ones			+		_____	_____
Tens	Ones																
+																	
_____	_____																
Tens	Ones																
+																	
_____	_____																

## Kinder – 42 control, 34 Experimental      Part. Eta squared .25

79.4% of the Kindergarten students who received intervention (and were thus below the 25<sup>th</sup> percentile in the fall) were above the benchmark (25<sup>th</sup> percentile) in the spring. Of that group, 67.6% of exper. students were above the 35<sup>th</sup> percentile in the spring.

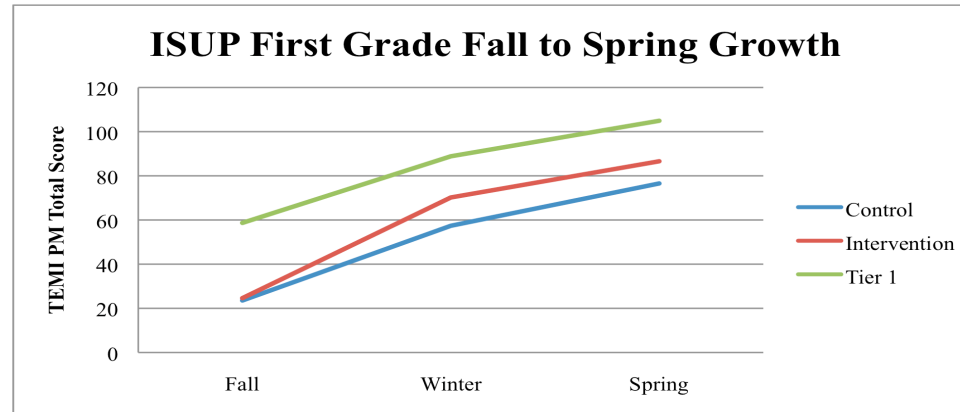


Preliminary findings - means for 3 groups across 3 time points; analyses being conducted

# First Grade – 61 Control; 49 Experimental

Part. Eta squared .05

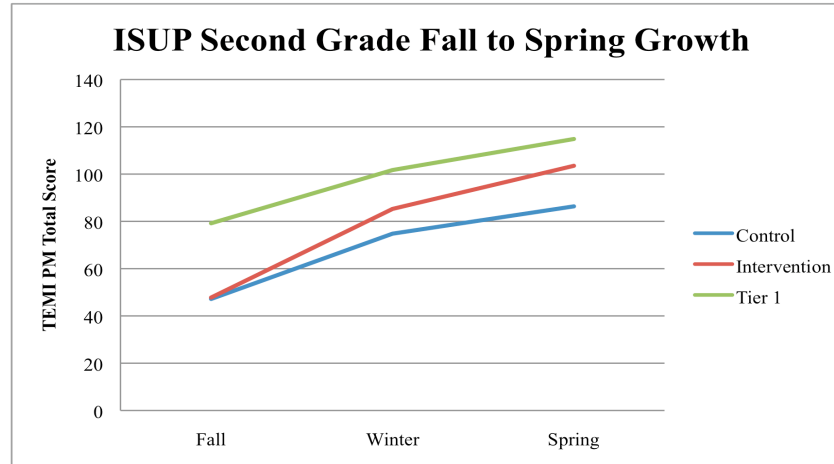
- 57.7% of the First Grade students who received intervention (and were thus below the 25<sup>th</sup> percentile in the fall) were above the benchmark (25<sup>th</sup> percentile) in the spring. Of that group, 50.0% of intervention students were above the 35<sup>th</sup> percentile in the spring.



Preliminary findings - means for 3 groups across 3 time points; analyses being conducted

## Second Grade – 82 control, 66 experimental Part. Eta squared .03

- 68.2% of the Second Grade students who received intervention (and were thus below the 25<sup>th</sup> percentile in the fall) were above the benchmark (25<sup>th</sup> percentile) in the spring. Of that group, 56.1% of intervention students were above the 35<sup>th</sup> percentile in the spring.



Growth curve models - tested constrained vs. unconstrained versions to come up with a final model

The slopes are sig. different for control vs. experimental, and for experimental vs. Tier 1, indicating that the experimental group is on a trajectory to catch up with Tier 1.

*Evidence-based Interventions for Students  
With Learning Disabilities:  
How Research Can Inform Practice  
Council for Learning Disabilities  
October 3, 2009*

*David Chard*

Dean of the Annette Caldwell Simmons School of Education &  
Human Development, Southern Methodist University

*Diane P. Bryant*

Project Director for the Meadows Center for Preventing Educational Risk:  
Mathematics Institute for Learning Disabilities & Difficulties



# What Works Clearinghouse

- **WWC Evidence Standards:**
  - identify studies that provide the strongest evidence of effects
  - randomized controlled trials and regression discontinuity studies, and secondarily quasi-experimental studies of especially strong design
  - "Meets Evidence Standards"
  - "Meets Evidence Standards with Reservations"
  - "Does Not Meet Evidence Screens"

<http://ies.ed.gov/ncee/wwc/>

<http://ies.ed.gov/ncee/wwc/reports/topic.aspx?tid=04> (reports)

# WWC: Under Review

- Programs under review
  - Accelerated Math
  - Bridges in Mathematics
  - Compass Learning Odyssey
  - Investigations in Number, Data, and Space
  - Kumon Mathematics Program

# The Access Center

- [http://www.k8accesscenter.org/training\\_resources/math.asp](http://www.k8accesscenter.org/training_resources/math.asp)
  - Mathematics Strategy Instruction (SI) for Middle School Students with Learning Disabilities
  - Using Mnemonic Instruction to Teach Math
  - Using Peer Tutoring for Math
  - Computer-Assisted Instruction and Math
  - Direct/Explicit Instruction and Math
  - Learning Strategies and Math
  - Concrete-Representational-Abstract Instructional Approach
  - Learner Accommodations and Instructional Modifications for Students with Learning Disabilities

# More Resources

- Math Differentiation Brief
- Math Graphic Organizers
- Math Problem Solving for Primary Elementary Students with Disabilities
- Math Problem Solving for Upper Elementary Students with Disabilities
- Illuminations <http://illuminations.nctm.org/>
- MathTools <http://www.mathforum.org/mathtools/>
- Meadows Center for Preventing Educational Risk: Mathematics Institute for Learning Disabilities and Difficulties
  - <http://www.meadowscenter.org/>
  - [www.earlymathintervention.org](http://www.earlymathintervention.org)